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14. ABSTRACT

Many scientific and engineering communities have recognized that intrinsic uncertainties in an engineering structural system have multifaceted natures (randomness, non-randomness, partial randomness, vagueness, and so forth) and traditional probability theory does not always provide an appropriate framework for handling the various types of uncertainties. The primary objectives of this research work were to explore the possible mathematical frameworks of Uncertainty Quantification (UQ) and to develop an appropriate and unified framework for multiple types of uncertainty sources. The candidate frameworks include probability theory and evidence theory (Dempster-Shafer theory). Multidisciplinary Optimization is addressed by developing sensitivity analysis of quantified uncertainty in a structural system and a cost-efficient UQ methodology.

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Uncertainty Quantification, Probability Theory, Evidence Theory, Multidisciplinary Optimization, Imprecise Information

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Final Report

Computational Mathematics for Determining Uncertain Bounds in Multi-Valued Engineering Design

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Research Objectives

Many scientific and engineering communities have recognized that intrinsic uncertainties in an engineering system have multifaceted nature (randomness, non-randomness, partial randomness, vagueness, and so forth) and traditional probability theory does not always provide an appropriate framework for handling various types of uncertainties, especially for a large-scale and complex engineering system, such as an aircraft.

The primary objectives of this research work were to explore the possible mathematical frameworks of Uncertainty Quantification (UQ) and to develop an appropriate and unified framework for multiple types of uncertainty sources. The candidate frameworks include possibility theory (Fuzzy Set theory), probability theory and evidence theory (Dempster-Shafer theory). To address the one of main difficulties in UQ, high computational cost, efficient methodologies were investigated and developed.

To achieve those objectives, following research goals were set:

1. Develop an efficient and accurate UQ methodology for engineering structural systems using possibility theory, evidence theory, and probability theory.
2. Develop sensitivity analysis techniques in UQ using evidence theory for identifying primary contributors for the propagated uncertainty in an engineering system.
3. Model a structural system immune to various uncertainties by using a design optimization technique with multidisciplinary system reliability requirements from evidence theory.
4. Investigate probabilistic UQ frameworks, Polynomial Chaos Expansion (PCE), and develop an efficient, non-intrusive formulation procedure using the Latin Hypercube Sampling and a statistical test procedure.
5. Identify where significant contributors of uncertainty occur in systems using the PCE technique.
6. Develop an efficient simulation method to represent random fields efficiently in the PCE technique.

Relevance to U.S. Air Force

In a high-performance aircraft structural design, as the complexity of mechanical systems and multidisciplinary performance requirements (including airframe flutter, fluid mechanics, controls, thermal-electrical-mechanical behavior and so forth) become large and stringent, it is imperative to take various uncertainties into consideration. Some uncertainties (loads, material properties and boundary conditions) occur randomly, and they can be modeled with well-known probability density functions. However, certain parameters cannot be assigned with any random function; they may take values within certain bounds due to the inherent non-randomness (for example, initial conditions or system failure modes). In conceiving high-performance mechanical systems, such as an innovative aircraft, it becomes obvious that our knowledge and data suffer from sheer imprecision because we must explore beyond the current level of technological knowledge and experience. One way to address these computational limitations in an aircraft structural design is to utilize Uncertainty Quantification (UQ) techniques. The U.S. Air Force could achieve analytical certification of the designed components or an entire system for their demanding performances by employing these UQ techniques.

Accomplishments/Findings

This research work started with the survey and investigation of current Uncertainty Quantification (UQ) techniques (Possibility theory, Evidence theory, and a probabilistic UQ technique—Polynomial Chaos Expansion) for engineering structural systems. The competence and limitations of these techniques were represented by classifying uncertainties into different types (aleatory and epistemic uncertainties) as shown in Figure 1.

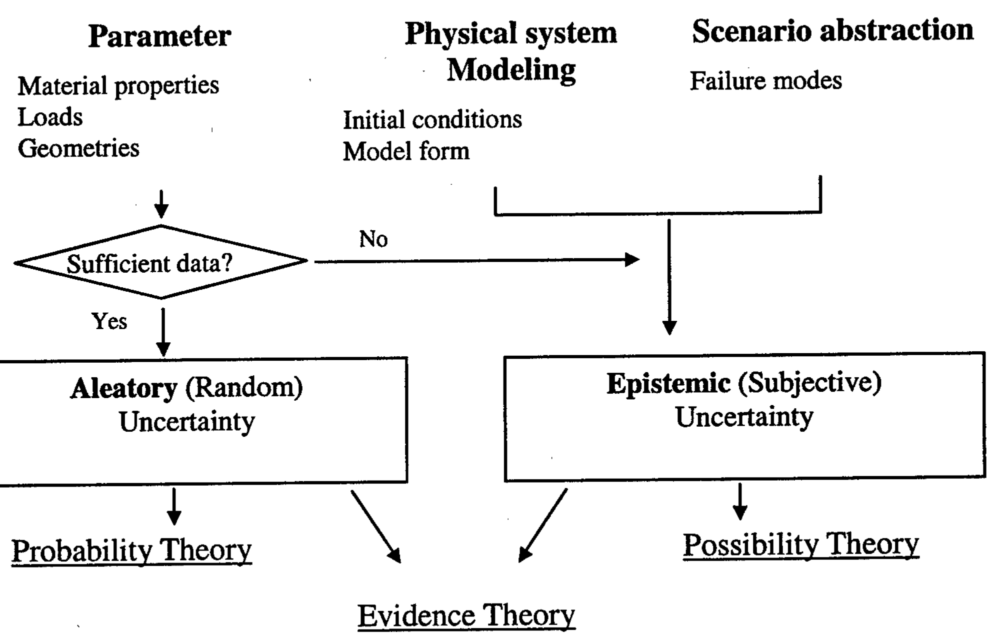


Figure 1. Solution approaches based on uncertainty type.

Aleatory uncertainty is also called irreducible or inherent uncertainty. Epistemic uncertainty is a subjective or reducible uncertainty that stems from lack of knowledge and data. Formal theories introduced to handle uncertainty include classical probability theory, possibility theory, and evidence theory. The common issue among these theories is how to determine the degree to which uncertain events are likely to occur. A distinct difference among these theories is in assignment of degree of belief. Both classical probability theory and evidence theory limit the total belief for all possible events to be unity. On the other hand, there is no such restriction in possibility theory, since one may have perfect confidence for a certain event and may give a possibility of unity through a possibility distribution.

1) Uncertainty Quantification using Evidence Theory

As a generalization of classical probability and possibility theories from the perspective of bodies of evidence and their measures, evidence theory can handle both epistemic and aleatory uncertainties in one framework. Evidence theory allows for pre-existing probability information to be utilized together with epistemic information (certain bounds or possibilistic membership function, etc) to assess likelihood for a limit-state function.

Accomplishments in Evidence Theory

- A general methodology for using evidence theory was developed for an engineering structural reliability analysis where only limited information is available and multiple types of uncertainty exist.
- The high computational cost in using evidence theory is reduced by employing a cost-effective algorithm using the Multiple Point Approximation (MPA) technique. A new system reanalysis method (Successive Matrix Inversion) was developed and applied to the preliminary design of an Intermediate Complexity Wing (ICW) structure.
- Sensitivity analyses of plausibility with respect to an expert opinion and structural deterministic parameters were analytically developed. The information from these sensitivity analyses can be used to improve the current design and to perform the data acquisition in a more efficient way.
- An optimization technique based on reliability analysis was investigated and developed to produce a robust engineering structure for intrinsic uncertainties and demonstrated with an example of multidisciplinary design optimization of an ICW.

Until now, when multiple types of uncertainties coexist in a target structural reliability analysis, UQ analyses have been performed by treating them separately or by making assumptions to accommodate either the probabilistic framework or the fuzzy set framework. Hence, the possibility of adopting evidence theory as a general tool of UQ analysis for multiple types of uncertainties was investigated. It was found that because of the flexibility of the basic axioms in evidence theory, not only aleatory (random) uncertainty, but also epistemic (non-random) uncertainty could be tackled in its framework without any baseless assumptions. The Basic Belief Assignment (BBA) structure in evidence theory usually is not a continuous explicit function for the given imprecise information. Because of the discontinuity in BBA, intensive computational cost might be inevitable in quantifying uncertainty using evidence theory.

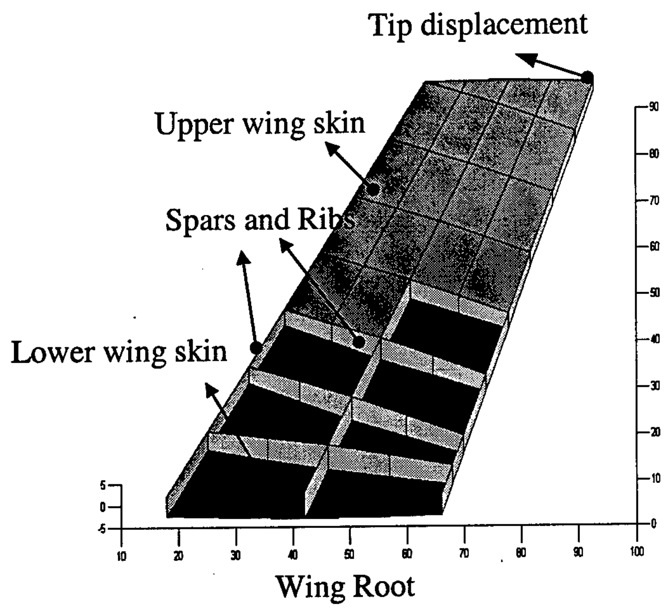


Figure 2. Intermediate Complexity Wing (ICW) structure model

To alleviate the intensive computational cost, a cost-effective algorithm using MPA was developed. In the algorithm, optimization and approximation techniques were employed to identify the failure region and invest our computational resources only on the identified failure region. It was found that the belief and plausibility functions were computed efficiently without sacrificing the accuracy of resulting measurements. The cost-effective algorithm was applied to the preliminary design of an Intermediate Complexity Wing (ICW) structure as shown in Figure 2. Root chord nodes are constrained as supports. Aerodynamic loads are applied along the wing surface.

Figure 3 shows the Complementary Cumulative Functions (CCFs) from different frameworks for an UQ example with the same multiple interval information. CCFs can be interpreted in the same way as the cumulative distribution function in probability theory. As shown in this figure, probability theory does not allow any impreciseness on the given information, so it gives a single valued result. However, possibility theory and evidence theory give a bounded result. The result from possibility theory gives the most conservative bound ([0, *Necessity*]), essentially because of the Zadeh's extension principle. In that principle, the degree of membership of the system response corresponds to the degree of membership of the overall most preferred set of fuzzy variables. Evidence theory gives an intermediate bounded result ([*Belief*, *Plausibility*]), which always includes the probabilistic result; that is, lower and upper bounds of probability based on the available information. It was found that a BBA structure in evidence theory can be used to model both fuzzy sets and probability distribution functions due to its flexibility. That is, different types of information (fuzzy membership function and PDF) can be incorporated in one framework to quantify uncertainty in a system.

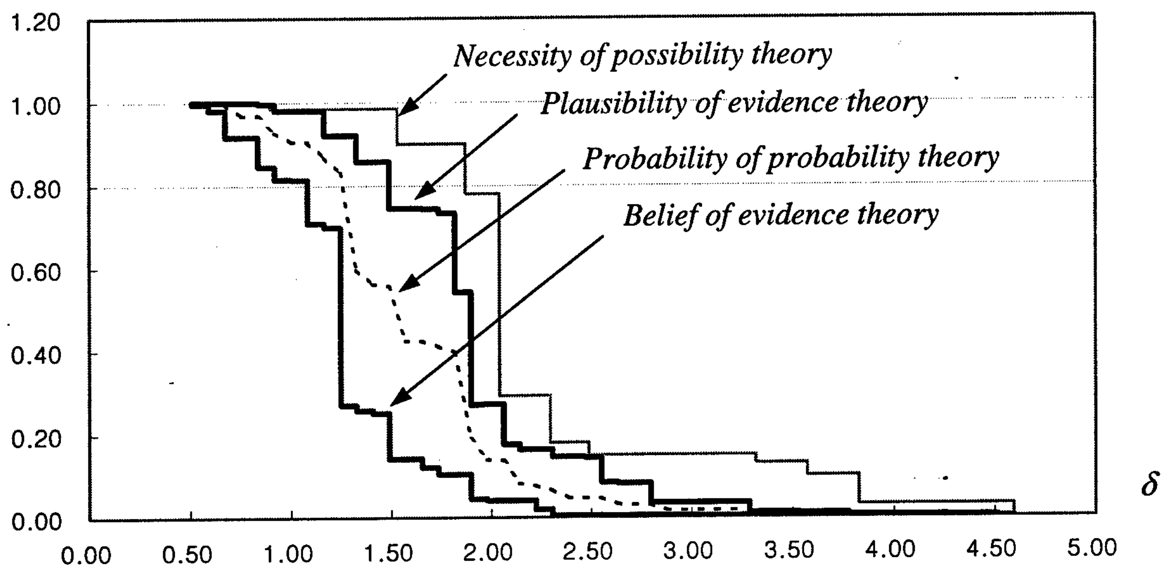


Figure 3. Complementary cumulative measurements of possibility theory, probability theory and evidence theory with multiple interval information

The obtained bounded result of evidence theory, which tends to be less conservative than that of possibility theory and less restrictive than the result of probability theory, can be viewed as the best estimate of system uncertainty because the given imprecise information is propagated through the given limit state function without any unnecessary assumptions in evidence theory.

In sensitivity analysis of plausibility with respect to an expert opinion, it was our goal to find the primary contributing expert opinion for the degree of plausibility. The result from sensitivity analysis indicates on which proposition the computational effort and future collection of information should be focused. This sensitivity analysis can be easily shifted from the sensitivity for plausibility to the sensitivity for uncertainty, which is defined by the subtraction of belief from plausibility. By decreasing the degree of uncertainty, we can be more confident in the reliability analysis result.

The sensitivity of a deterministic parameter in an engineering structural system was developed to improve the current design by decreasing the failure plausibility of a limit-state function efficiently. However, the plausibility function in evidence theory is a discontinuous function for varying values of a deterministic parameter because of the discontinuity of a BBA structure for uncertain parameters. The gradient of plausibility was represented using the degree of plausibility decision (*Pl_dec*), which was introduced by employing the generalized insufficient reason principle to the plausibility function. *Pl_dec* can be used as a supplemental measurement to make a decision as to whether a system can be accepted.

For high efficiency design of engineering structures, mathematical optimization techniques are usually employed. However, without considering the uncertainty in design parameters, operating conditions, and physical behavior, the optimized design might result in a catastrophically high risk. Hence, in conjunction with the cost-effective algorithm and sensitivity techniques, evidence theory was applied to the design optimization based on reliability analysis. The resulting design of a target structure has a robust optimal performance for the intrinsic uncertainties.

2) Uncertainty Quantification using Possibility Theory

In structural engineering it is often possible to acquire knowledge about various parameters in the form of low, probable, and high values. Based on this information, the membership functions can be constructed in possibility theory. Following the concepts of possibility theory (fuzzy set theory), the parameters are modeled as fuzzy numbers where the information is imprecise due to vague information. The present work is directed towards the development of a method that is capable of handling the information available as fuzzy numbers.

Accomplishments in Possibility Theory

- Uncertainty quantification for non-random uncertainty and the non-monotonic limit state function was investigated by using possibility theory.
- For the estimation of the fuzzy membership function, a numerical analysis-based methodology was developed and demonstrated with various engineering structures, including a fighter wing model.
- A methodology of employing nonlinear function approximations and a D-optimal design technique was developed to reduce the computational effort involved in the solution process.
- Optimization algorithms were investigated to obtain designs that are the least sensitive to input uncertainty of those available as a possibility distribution.

The problem dealt with here is the estimation of the membership function of the response subject to uncertain input parameters, which are bounds on the response at various confidence levels. The uncertainty is non-random, and it is defined using possibility theory. The estimation of the fuzzy membership function for the implicit response requires the use of interval analysis at each confidence level. The interval analysis techniques available in the literature use linear approximation at the central values or evaluate the function at all the vertices formed by the lower and upper limits of the uncertain variables. These methods require a significant number of function evaluations and sometimes fail to capture the bounds of response for non-monotonic response functions. Therefore, a numerical analysis-based method using nonlinear function approximations to reduce the computational effort involved in the solution process was developed.

Figure 4 shows the structural model of the flexible wing whose membership function (bounds on the response at various confidence levels) for the frequency response is considered. The structure represents a fighter wing model with all the dynamic characteristics. The first natural frequency of the structure is considered as the response in this example.

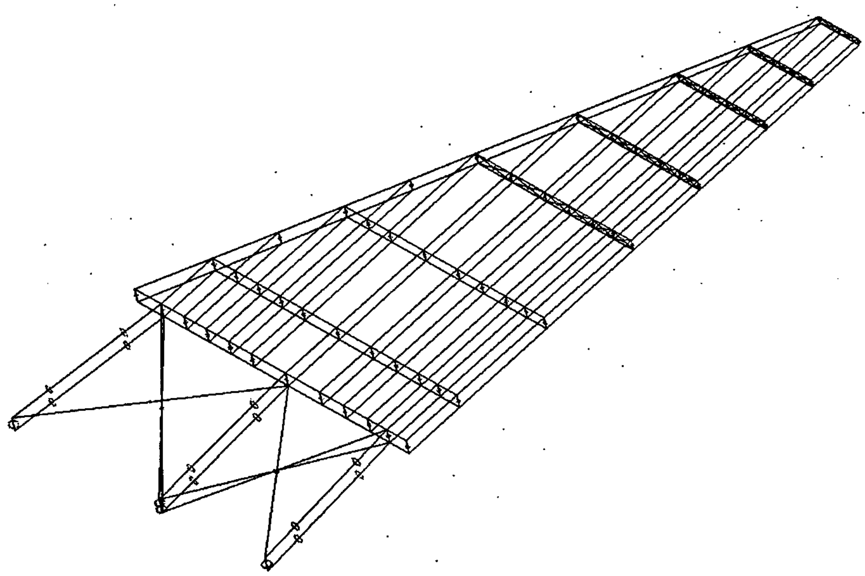


Figure 4. Flexible wing structural model

D-optimal design technique was used to select multiple design points to construct a Multi Point Approximation (MPA) for the response function. Local TANA2 approximations were constructed using the data points, and they were blended using a blending function to obtain one MPA. Once the approximation is constructed, random sampling is performed in the uncertain space, and the maximum and minimum for the function are determined. These maximum and minimum points are perturbed, and a new simulation is performed at the perturbed data points. This additional information is used in constructing a new TANA2 approximation that is used to determine the lower and upper limits of the response.

These lower and upper limits are obtained at each of the possibility levels to produce the membership function of the response as shown in Figure 5. The membership function determines the bounds on the response at various confidence levels. The uncertainty in the input data would result in the bounded response, depending on the bounds of input parameters. The possibility of failure is equal to the maximum value of the confidence level in the range of response values considered. The response membership function describes the relationship between the possibility level and frequency. The membership function can give information about the possibility for a range of frequencies. In this technique, the structure can be designed to operate in the range of response values that satisfy certain confidence level requirements.

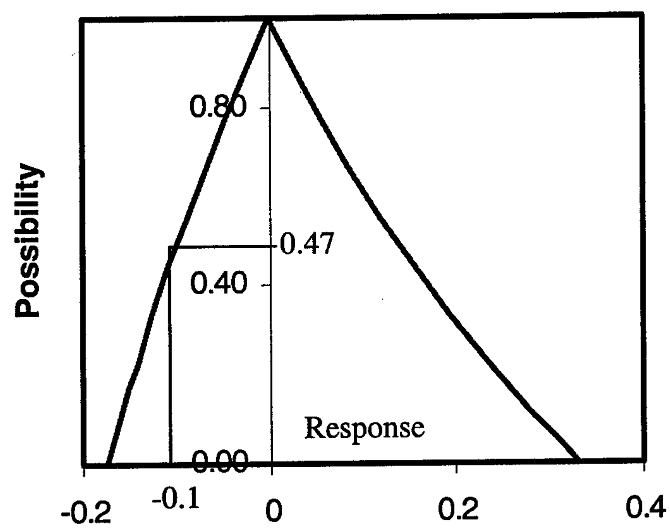


Figure 5: Bounds on Frequency Response

The proposed possibilistic analysis method is an efficient technique to estimate the possibility of failure of the response with vague information. The efficiency of the method is achieved by using high fidelity surrogate models for the response.

Due to the non-deterministic nature of design parameters and operating environments, design optimization without including uncertainty would result in potentially high-risk designs. This is because the final design point is usually located on active constraints that can easily be violated because of the variance in the design parameters. Traditionally, the risk involved in this type of design process was handled by using safety factors, which generate non-optimal products without any insight into the uncertainties in the design. In this research work, with the developed efficient methodologies of possibility theory, an optimization algorithm was investigated for a robust and reliable performance bound with a certain confidence level.

3) Uncertainty Quantification using Probability Theory

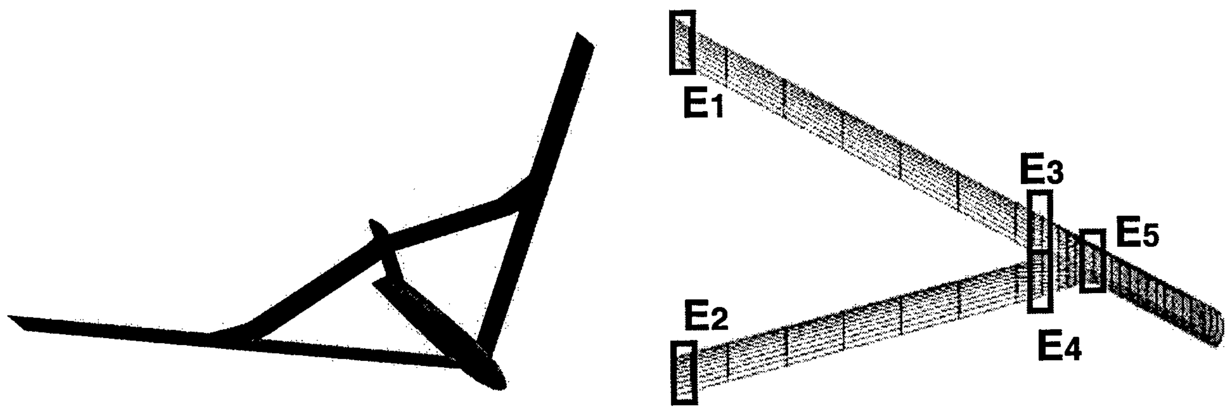
Polynomial Chaos Expansion (PCE) is one efficient choice for uncertainty analysis using probability theory with complete and sufficient data and information. Stochastic expansions provide analytically appealing convergence properties based on the concept of a random process. Stochastic expansions can be applied in two types of UQ using probability theory: the non-intrusive formulation and intrusive formulation (Stochastic Finite Element Method) procedures. The non-intrusive formulation procedure means that the stochastic expansion is employed to create the response surface as a surrogate model without interfering with the Finite Element Method (FEM) procedure. By contrast, the use of Polynomial Chaos Expansion (PCE) to directly modify the stiffness matrix of an FEM procedure is the intrusive formulation method.

Accomplishments in Probability Theory

- To deal with the random nature of input parameters of structural models, several efficient probabilistic methods were investigated and developed. Specifically, the non-intrusive formulation, consisting of the polynomial chaos expansion and Latin Hypercube Sampling, was used to represent the response of an uncertain system.
- To handle non-normal random variables in the non-intrusive formulation, the generalized PCE algorithm and transformation technique were investigated and implemented.
- To specify the probability of the UQ results using PCE, the confidence interval interpretation of the mean response was successfully derived and conducted for complex structural systems.
- To represent random fields, an efficient representation scheme, the Karhunen-Loeve transformation, was introduced to Latin Hypercube Sampling.

In our research work, the non-intrusive formulation, specifically utilizing PCE, was selected, since this approach can reduce the computational effort in large-scale engineering design applications. This study presented a computationally efficient procedure for quantifying uncertainty and finding significant parameters of uncertainty models.

An implementation of PCE for different probability distributions was also conducted. Two existing techniques, a generalized PCE algorithm and the transformation technique, were investigated and incorporated for non-normal random variable cases. Several analytical examples and a highly nonlinear structural model of an uninhabitated joined-wing aircraft (Figure 6) were used to verify the effectiveness and applicability of this proposed procedure. The Young's moduli for these groups are denoted by E_1 through E_5 as shown in Figure 6b. The Young's moduli of five locations were modeled as uncorrelated random variables using Gaussian distributions. The particular design issue of the joined wing is how to overcome the buckling failure of the aft wing and the corresponding aeroelastic complications under the presence of compressive loads. The critical aeroelastic responses, such as flutter and divergence, may become coupled with the global and post buckling response of the lifting surfaces of the joined wing. Therefore, we conducted a buckling analysis of the joined-wing model based on the non-intrusive formulation procedure by evaluating the undetermined coefficients of the PCE through LHS.



(a) Joined wing sensorcraft concept (b) Locations of five random variable parts

Figure 6. Stochasticity in Joined Wing

Our research developed a computationally efficient procedure for quantifying uncertainty and illustrated applicability to a large computational model (i.e. joined wing aircraft). In the developed framework, combining several modules relevant to probabilistic methods provides a computationally efficient procedure for uncertainty analysis. An efficient sampling scheme, LHS, was employed and improved for evaluating the generalized Fourier coefficients of the polynomial chaos expansion. Since the key challenge in uncertainty analysis is to find the most significant components that drive response variability, analysis of variance and residual analysis were conducted to find the significant parameters in UQ. Many practical engineering situations require a wide variety of skewed distributions, which are not bell-shaped and symmetric like the normal distribution. To cope with these non-normal distributions, two existing techniques, a generalized PCE algorithm and the transformation technique, were investigated and verified in terms of the accuracy and efficiency. Several structural models were used to demonstrate the non-intrusive formulation procedure of PCE for the normal and non-normal distributions using these two techniques. In addition, the confidence interval interpretation of the mean response of the joined wing was successfully conducted. By incorporating the confidence interval from previous information, the engineer can ensure that the mean response will lie inside the interval before any additional experiments and data of a target system is acquired.

In practice for UQ in engineering systems, many uncertainties in geometric or material properties such as Young's modulus are spatially correlated over the structure. Hence the spatial variability of mechanical properties of structural systems must be considered by using the concept of the random field. The random field realizations can be obtained efficiently through the Karhunen-Loeve transformation (KLT) within the framework of LHS. The proposed methodology with both LHS and KLT can lead to a considerable reduction of simulation costs of complex and computationally intensive problems compared to the previous PCE techniques. Moreover, the proposed methodology helps to identify the significant parameters of the uncertainty model and suggests efficient representation schemes for random fields and non-normal random variables. Once the PCE formulation is constructed, there is no extra computational cost to obtain the statistical properties of the responses. Since the statistical estimation of how much a given uncertain input drives the risk of the system is obtained analytically in the proposed procedure, better decisions can be made for robust and improved performances of a target system. Therefore, The presented approach can be a valuable tool in UQ in cases where the investigation of significant parameters of the stochastic model is critical and the deterministic tools do not reflect the stochastic nature of system behavior.

In summary, UQ in engineering systems is scientifically investigated by addressing the propagation of various types of uncertain sources. To predict crucial system performances with non-deterministic physical factors accurately, different frameworks from probability theory, possibility theory and evidence theory were investigated with airframe structures. Due to the nature of many repetitive analyses of UQ in engineering systems, one of the main issues in this research work was to develop efficient methodologies. Many useful and efficient methodologies and techniques, such as bounded UQ framework, a cost-efficient algorithm, sensitivity analysis techniques, an efficient possibilistic UQ technique, improved probabilistic random field technique and so forth, were developed and demonstrated with several practical examples. These techniques are particularly useful and vital for the analytical certification of the performances of an aircraft that is subject to unpredicted and variable mission environments.

Personnel Supported

The following Ph.D. students were supported on this research program during the award period: Dr. Ravi Penmetsa, Mr. Ha-Rok Bae and Mr. Seung-Kyum Choi.

Interactions/Transitions

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Meetings, Conferences and Symposium Presentations

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Patent Disclosures

None

Honors/Awards

"*University Professor Title*" awarded to Dr. Grandhi at Wright State University in 1998.

"*Distinguished Professor of Research*" awarded to Dr. Grandhi for the period 1996 – 2006.

"*Young Researcher Fellowship Award* for exemplary research in computational mechanics" awarded to Penmetsa, Ravi by First MIT conference on Computational Fluid ad Solid Mechanics, June 2001.

"*Outstanding Graduate Student Award*" awarded to Bae, Ha-Rok by Wright State University, May 2003.

"*Young Researcher Fellowship Award* for exemplary research in computational mechanics" awarded to Bae, Ha-Rok by Second MIT conference on Computational Fluid ad Solid Mechanics, June 2003.

"*Young Researcher Fellowship Award* for exemplary research in computational mechanics" awarded to Choi, Seung-Kyum by Second MIT conference on Computational Fluid ad Solid Mechanics, June 2003.

"*Second Best Paper Award*" awarded to Bae, Ha-Rok by International Air and Space Symposium: The next 100 years, Dayton, OH, July 2003.